

# Leptogenesis without $CP$ violation at low energies

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In this paper we give a class of examples where the decays of the heavy Majorana neutrinos may violate  $CP$  even if there is no  $CP$  violation at low energies, i.e., where leptogenesis can take place without Majorana- or Dirac-type  $CP$  phases at low energies.

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There is strong experimental evidence for neutrino oscillations [1,2], thus implying non-zero neutrino masses and mixing in the leptonic sector, together with the possibility of  $CP$  violation. The most straightforward way of extending the standard model (SM) in order to incorporate neutrino masses is to add one neutrino field per generation, singlet under the  $SU(3)_c \times SU(2) \times U(1)$  gauge symmetry, in analogy with the quark sector. The fact that neutrinos are neutral particles allows for the introduction of a Majorana mass term for the right-handed gauge singlets together with the usual Dirac mass term, provided that lepton-number conservation is not imposed. This leads to the seesaw mechanism [3], which accounts in an elegant and simple way for the smallness of neutrino masses. Leptonic  $CP$  violation may play a crucial rôle in the generation of the observed baryon number asymmetry of the universe (BAU) via leptogenesis. In this framework a  $CP$  asymmetry is generated through out-of-equilibrium  $L$ -violating decays of heavy Majorana neutrinos [4] leading to a lepton asymmetry  $L \neq 0$  while  $B=0$  is still maintained. Subsequently, sphaleron processes [5], which are  $(B+L)$ -violating and  $(B-L)$ -conserving, restore  $(B+L)=0$ , thus creating a nonvanishing  $B$ . Several groups have analyzed the requirements on models leading to a viable leptogenesis [6]. At low energies the decoupling limit is an excellent approximation and there are three  $CP$ -violating phases in the corresponding mixing matrix, one of Dirac type, which could be observed in neutrino oscillations [7], and two of Majorana type, which can be interpreted in terms of unitary triangles [8]. The question of whether or not it is possible to establish a connection between leptogenesis and  $CP$  violation at low energies is very interesting and has been addressed by several authors [9,10]. It has been shown that, although in general this connection cannot be established, there are several frameworks where the sign and size of the observed baryon asymmetry obtained through leptogenesis can be related to  $CP$  violation at low energies.

It is well known that in the case of three generations with no left-handed Majorana mass term there are six  $CP$ -violating phases in the leptonic sector [11]. It is possible to choose a Weak Basis (WB) where all of these phases only appear in the Dirac-type neutrino mass matrix. These phases

may be parametrized in such a way that the three low-energy  $CP$ -violating phases are a function of all of them whilst leptogenesis can be written in terms of only three phases [10].

In this work we want to emphasize that leptogenesis can take place even if there is no  $CP$  violation at low energies. The prospects of finding  $CP$ -violating effects at low energies, for instance in future neutrino factories, are extremely exciting; yet it is important to notice that leptogenesis remains in principle a viable scenario even if no  $CP$  violation is seen at low energies.

## FRAMEWORK

After spontaneous symmetry breaking, the leptonic mass term for a minimal extension of the SM, which consists of adding to the standard spectrum one right-handed neutrino per generation, can be written as

$$\begin{aligned} \mathcal{L}_m = & - \left[ \bar{\nu}_L^0 m \nu_R^0 + \frac{1}{2} \nu_R^{0T} C M \nu_R^0 + \bar{l}_L^0 m_l l_R^0 \right] + \text{H.c.} \\ = & - \left[ \frac{1}{2} n_L^T C \mathcal{M}^* n_L + \bar{l}_L^0 m_l l_R^0 \right] + \text{H.c.}, \end{aligned} \quad (1)$$

where  $m$ ,  $M$ , and  $m_l$  denote the neutrino Dirac mass matrix, the right-handed neutrino Majorana mass matrix, and the charged lepton mass matrix, respectively, and  $n_L = (\nu_L^0, (\nu_R^0)^c)$ . In this minimal extension of the SM a term of the form  $\frac{1}{2} \nu_L^{0T} C m_L \nu_L^0$  does not appear in the Lagrangian and the matrix  $\mathcal{M}$  is given by

$$\mathcal{M} = \begin{pmatrix} 0 & m \\ m^T & M \end{pmatrix} \quad (2)$$

with a zero entry on the (11) block. The right-handed Majorana mass term is  $SU(2) \times U(1)$  invariant; consequently it can have a value much above the scale  $v$  of the electroweak symmetry breaking, thus leading to the see-saw mechanism. The neutrino mass matrix  $\mathcal{M}$  is diagonalized by the transformation

$$V^T \mathcal{M}^* V = \mathcal{D}, \quad (3)$$

where  $\mathcal{D} = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, M_{\nu_1}, M_{\nu_2}, M_{\nu_3})$ , with  $m_{\nu_i}$  and  $M_{\nu_i}$  denoting the physical masses of the light and heavy Majorana neutrinos, respectively. It is convenient to write  $V$  and  $\mathcal{D}$  in the following form:

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$$V = \begin{pmatrix} K & R \\ S & T \end{pmatrix}, \quad \mathcal{D} = \begin{pmatrix} d & 0 \\ 0 & D \end{pmatrix}. \quad (4)$$

From Eq. (3) one obtains, to an excellent approximation,

$$-K^\dagger m \frac{1}{M} m^T K^* = d, \quad (5)$$

together with the following exact relation:

$$R = m T^* D^{-1}. \quad (6)$$

In the WB where the right-handed Majorana neutrino mass is diagonal, it also follows to an excellent approximation that

$$R = m D^{-1}. \quad (7)$$

Equation (5) is the usual seesaw formula with  $K$  a unitary matrix. The neutrino weak-eigenstates are related to the mass eigenstates by

$$\nu_{iL}^0 = V_{i\alpha} \nu_{\alpha L} = (K, R) \begin{pmatrix} \nu_{iL} \\ N_{iL} \end{pmatrix} \quad \begin{pmatrix} i=1,2,3 \\ \alpha=1,2,\dots,6 \end{pmatrix}, \quad (8)$$

and thus the leptonic charged-current interactions are given by

$$-\frac{g}{\sqrt{2}} (\bar{l}_{iL} \gamma_\mu K_{ij} \nu_{jL} + \bar{l}_{iL} \gamma_\mu R_{ij} N_{jL}) W^\mu + \text{H.c.} \quad (9)$$

From Eqs. (8), (9) it follows that  $K$  and  $R$  give the charged-current couplings of charged leptons to the light neutrinos  $\nu_j$  and to the heavy neutrinos  $N_j$ , respectively. The unitary matrix  $K$ , which contains all the information about  $CP$  violation at low energies, can be parametrized as

$$K = P_\xi \hat{U}_\rho P_\alpha \rightarrow \hat{U}_\rho P_\alpha \quad (10)$$

with  $P_\xi = \text{diag}(\exp(i\xi_1), \exp(i\xi_2), \exp(i\xi_3))$ , and  $P_\alpha = \text{diag}(1, \exp(i\alpha_1), \exp(i\alpha_2))$  leaving  $\hat{U}_\rho$  with only one phase as in the case of the Cabibbo, Kobayashi and Maskawa matrix. Since  $P_\xi$  can be rotated away by a redefinition of the charged leptonic fields,  $K$  is left with three  $CP$ -violating phases, one of Dirac type  $\rho$  and two of Majorana character  $\alpha_1$  and  $\alpha_2$ .

The computation of the lepton-number asymmetry, in this extension of the SM, resulting from the decay of a heavy Majorana neutrino  $N^j$  into charged leptons  $l_i^\pm$  ( $i=e, \mu, \tau$ ) leads to [12]

$$A^j = \frac{g^2}{M_W^2} \sum_{k \neq j} \left[ \text{Im}((m^\dagger m)_{jk} (m^\dagger m)_{jk}) \frac{1}{16\pi} \right. \\ \left. \times \left( I(x_k) + \frac{\sqrt{x_k}}{1-x_k} \right) \right] \frac{1}{(m^\dagger m)_{jj}}$$

$$= \frac{g^2}{M_W^2} \sum_{k \neq j} \left[ (M_k)^2 \text{Im}((R^\dagger R)_{jk} (R^\dagger R)_{jk}) \frac{1}{16\pi} \right. \\ \left. \times \left( I(x_k) + \frac{\sqrt{x_k}}{1-x_k} \right) \right] \frac{1}{(R^\dagger R)_{jj}}, \quad (11)$$

with the lepton-number asymmetry from the  $j$  heavy Majorana particle,  $A^j$ , defined in terms of the family number asymmetry  $\Delta A_i^j = N_i^j - \bar{N}_i^j$  by

$$A^j = \frac{\sum_i \Delta A_i^j}{\sum_i (N_i^j + \bar{N}_i^j)}. \quad (12)$$

The sum in  $i$  runs over the three flavors  $i=e, \mu, \tau$ ,  $M_k$  are the heavy neutrino masses, the variable  $x_k$  is defined as  $x_k = M_k^2/M_j^2$  and  $I(x_k) = \sqrt{x_k} [1 + (1+x_k) \log(x_k/(1+x_k))]$ . From Eq. (11) it can be seen that the lepton-number asymmetry is only sensitive to the  $CP$ -violating phases appearing in  $m^\dagger m$  in the WB, where  $M$  and  $m_l$  are diagonal (or equivalently in  $R^\dagger R$ ).

### LEPTOGENESIS WITH NO $CP$ VIOLATION AT LOW ENERGIES

Let us go to the WB where  $M$  and  $m_l$  are diagonal, real and positive ( $M \equiv D$ ) and choose a matrix  $m$  of the form [13]

$$m = i \hat{U}_\rho P_\alpha \sqrt{d} O^c \sqrt{D}, \quad (13)$$

where  $\sqrt{d}$  and  $\sqrt{D}$  are diagonal real matrices such that  $\sqrt{d} \sqrt{d} = d$ ,  $\sqrt{D} \sqrt{D} = D$  and  $O^c$  is an orthogonal complex matrix, i.e.  $O^c O^{cT} = 1$  but  $O^c O^{c\dagger} \neq 1$ . In this WB all  $CP$ -violating phases appear in  $m$ . From Eq. (13) together with Eq. (5) we obtain the matrix  $K$  given by Eq. (10) and in general it will violate  $CP$ . The physical relevance of this expression for the study of viable leptogenesis and its connection with low energy physics was emphasized by I. Masina at SUSY02 [14]. Particularizing for  $\alpha_1 = \alpha_2 = 0$  together with  $\rho = 0$ , there is no  $CP$  violation at low energies. Yet leptogenesis is sensitive to the combination  $m^\dagger m$ , which is given by

$$h \equiv m^\dagger m = \sqrt{D} O^{c\dagger} d O^c \sqrt{D}, \quad (14)$$

consequently, provided that the combination  $O^{c\dagger} d O^c$  is  $CP$ -violating, we may have leptogenesis even without  $CP$  violation at low energies either of Dirac or Majorana type.

It is possible to write WB-invariant conditions which have to vanish in order for  $CP$  invariance to hold. The non-vanishing of any of these invariants signals  $CP$  violation [15]. In Ref. [10] the following WB invariants, sensitive to  $CP$ -violating phases that appear in leptogenesis, were derived:

$$\begin{aligned}
I_1 &\equiv \text{ImTr}[hHM^*h^*M] = M_1M_2(M_2^2 - M_1^2)\text{Im}(h_{12}^2) \\
&\quad + M_1M_3(M_3^2 - M_1^2)\text{Im}(h_{13}^2) + M_2M_3 \\
&\quad \times (M_3^2 - M_2^2)\text{Im}(h_{23}^2), \\
I_2 &\equiv \text{ImTr}[hH^2M^*h^*M] = M_1M_2(M_2^4 - M_1^4)\text{Im}(h_{12}^2) \\
&\quad + M_1M_3(M_3^4 - M_1^4)\text{Im}(h_{13}^2) + M_2M_3 \\
&\quad \times (M_3^4 - M_2^4)\text{Im}(h_{23}^2), \tag{15}
\end{aligned}$$

$$\begin{aligned}
I_3 &\equiv \text{ImTr}[hH^2M^*h^*MH] = M_1^3M_2^3(M_2^2 - M_1^2)\text{Im}(h_{12}^2) \\
&\quad + M_1^3M_3^3(M_3^2 - M_1^2)\text{Im}(h_{13}^2) + M_2^3M_3^3 \\
&\quad \times (M_3^2 - M_2^2)\text{Im}(h_{23}^2).
\end{aligned}$$

The second equality for each  $I_i$  corresponds to the evaluation of these WB invariants in the WB where the right-handed neutrino mass is diagonal, with  $M_i$  the corresponding diagonal elements. The matrix  $H$  is defined by  $H \equiv M^\dagger M$ .

Choosing the matrix  $O^c$  of the form

$$O^c = A_{12} \cdot A_{23} \cdot A_{13} \tag{16}$$

with

$$\begin{aligned}
A_{12} &= \begin{pmatrix} \cosh \theta_{12} & i \sinh \theta_{12} & 0 \\ -i \sinh \theta_{12} & \cosh \theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\
A_{13} &= \begin{pmatrix} \cosh \theta_{13} & 0 & i \sinh \theta_{13} \\ 0 & 1 & 0 \\ -i \sinh \theta_{13} & 0 & \cosh \theta_{13} \end{pmatrix}, \tag{17} \\
A_{23} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cosh \theta_{23} & i \sinh \theta_{23} \\ 0 & -i \sinh \theta_{23} & \cosh \theta_{23} \end{pmatrix},
\end{aligned}$$

we obtain (simplifying the notation with cosh and sinh replaced by ch and sh):

$$\begin{aligned}
\text{Re } h_{12} &= (\text{sh}^2 \theta_{12} \text{sh} \theta_{13} \text{sh} \theta_{23} \text{ch} \theta_{23} d_1 \\
&\quad + \text{sh} \theta_{13} \text{sh} \theta_{23} \text{ch}^2 \theta_{12} \text{ch} \theta_{23} d_2 \\
&\quad + \text{sh} \theta_{13} \text{sh} \theta_{23} \text{ch} \theta_{23} d_3) \sqrt{M_1} \sqrt{M_2}, \\
\text{Im } h_{12} &= i \text{sh} \theta_{12} \text{ch} \theta_{12} \text{ch} \theta_{13} \text{ch} \theta_{23} \\
&\quad \times (d_1 + d_2) \sqrt{M_1} \sqrt{M_2},
\end{aligned}$$

$$\text{Re } h_{13} = -\text{sh} \theta_{12} \text{sh} \theta_{23} \text{ch} \theta_{12} (d_1 + d_2) \sqrt{M_1} \sqrt{M_3},$$

$$\begin{aligned}
\text{Im } h_{13} &= i[(\text{sh} \theta_{13} \text{ch}^2 \theta_{12} \text{ch} \theta_{13} \\
&\quad + \text{sh}^2 \theta_{12} \text{sh} \theta_{13} \text{sh}^2 \theta_{23} \text{ch} \theta_{13}) d_1 \\
&\quad + (\text{sh}^2 \theta_{12} \text{sh} \theta_{13} \text{ch} \theta_{13} \\
&\quad + \text{sh} \theta_{13} \text{sh}^2 \theta_{23} \text{ch}^2 \theta_{12} \text{ch} \theta_{13}) d_2 \\
&\quad + \text{sh} \theta_{13} \text{ch} \theta_{13} \text{ch}^2 \theta_{23} d_3] \sqrt{M_1} \sqrt{M_3},
\end{aligned}$$

$$\text{Re } h_{23} = \text{sh} \theta_{12} \text{sh} \theta_{13} \text{ch} \theta_{12} \text{ch} \theta_{23} (d_1 + d_2) \sqrt{M_2} \sqrt{M_3},$$

$$\begin{aligned}
\text{Im } h_{23} &= i(\text{sh}^2 \theta_{12} \text{sh} \theta_{23} \text{ch} \theta_{13} \text{ch} \theta_{23} d_1 \\
&\quad + \text{sh} \theta_{23} \text{ch}^2 \theta_{12} \text{ch} \theta_{13} \text{ch} \theta_{23} d_2 \\
&\quad + \text{sh} \theta_{23} \text{ch} \theta_{13} \text{ch} \theta_{23} d_3) \sqrt{M_2} \sqrt{M_3}. \tag{18}
\end{aligned}$$

The  $d_i$  are the diagonal elements of the matrix  $d$  (the masses of the light neutrinos). In general  $\text{Im } h_{ij}^2$ ,  $i \neq j$  do not vanish and there is leptogenesis. On the other hand, if any of the  $\theta_{ij}$  is zero, these imaginary parts vanish since all products  $\text{Re } h_{ij} \text{Im } h_{ij}$  contain the factor  $\text{sh} \theta_{12} \text{sh} \theta_{13} \text{sh} \theta_{23}$ . Equations (15) are no longer useful to discuss  $CP$  violation in the limit  $M_1 = M_2 = M_3$  since in this case they vanish trivially, although this degeneracy does not necessarily imply  $CP$  conservation at high energies [10].

## FINAL ADDITIONAL COMMENTS

In this framework low-energy physics only enters Eq. (14) through the masses of the light neutrinos, which are already constrained by experiment. In fact, Eq. (14) has no explicit dependence on mixing and  $CP$  violation at low energies, since  $K$  cancels out. With the present experimental knowledge there is freedom in the choice of the masses of the heavy neutrinos. Furthermore, low-energy physics is insensitive to the matrix  $O^c$ . As a result one can only establish a connection between leptogenesis and  $CP$  violation at low energies in models where additional constraints are imposed, so that, for instance, the matrix  $D$  is no longer independent of  $K$ .

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